GCE Examinations

Advanced Subsidiary / Advanced Level

Decision Mathematics
Module D2

Paper D

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.



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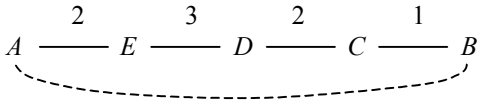
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D2 Paper D – Marking Guide

1. *(a)*

order:	1	5	4	3	2
	A	*B*	*C*	*D*	*E*
A	–	4	7	8	2
B	4	–	[1]	5	6
C	7	1	–	[2]	7
D	8	5	2	–	[3]
E	[2]	6	7	3	–

M1



A1

upper bound = 2 × weight of MST
= 2 × (2 + 3 + 2 + 1) = 2 × 8 = 16 miles — M1 A1

(b) use *AB* saving 2 + 3 + 2 + 1 – 4 = 4 — M1
new upper bound = 16 – 4 = 12 miles — A1 **(6)**

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2. *(a)* adding 5 to all entries to make them positive gives — M1

		B		
		I	II	III
A	I	11	1	4
	II	3	10	8
	III	10	6	2

new value of game $v = V + 5$ — A1

(b) let *B* play strategies I, II and III with proportions p_1, p_2 and p_3 — M1
let $x_1 = \frac{p_1}{v}$, $x_2 = \frac{p_2}{v}$, $x_3 = \frac{p_3}{v}$ — A1

(c) $p_1 + p_2 + p_3 = 1$ — M1
dividing by v gives $x_1 + x_2 + x_3 = \frac{1}{v}$
we wish to minimise v ∴ maximise $\frac{1}{v}$
objective function is maximise $P = x_1 + x_2 + x_3$ — A1

(d) from *A* I, $11p_1 + p_2 + 4p_3 \le v$
from *A* II, $3p_1 + 10p_2 + 8p_3 \le v$ — M1
from *A* III, $10p_1 + 6p_2 + 2p_3 \le v$

dividing by v gives the constraints

$11x_1 + x_2 + 4x_3 \le 1$
$3x_1 + 10x_2 + 8x_3 \le 1$
$10x_1 + 6x_2 + 2x_3 \le 1$ — A1 **(8)**
also $x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0$

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3. *(a)*

order:	1	6	2	3	5	4	7
	A	*B*	*C*	*D*	*E*	*F*	*G*
A	–	83	57	68	103	91	120
B	83	–	78	63	41	82	52
C	57	78	–	37	59	63	74
D	68	63	37	–	60	52	62
E	103	41	59	60	–	48	51
F	91	82	63	52	48	–	77
G	120	52	74	62	51	77	–

M1 A1

tour: *ACDFEBGA* A1

upper bound = 57 + 37 + 52 + 48 + 41 + 52 + 120 = 407 miles A1

(b) e.g. starting at *B*

order:		1	6	5	2	3	4
	A	*B*	*C*	*D*	*E*	*F*	*G*
A	–	83	57	68	103	91	120
B	83	–	78	63	41	82	52
C	57	78	–	37	59	63	74
D	68	63	37	–	60	52	62
E	103	41	59	60	–	48	51
F	91	82	63	52	48	–	77
G	120	52	74	62	51	77	–

M1 A1

lower bound = weight of MST + two edges of least weight from *A*
= (41 + 48 + 51 + 52 + 37) + 57 + 68 = 354 miles M1 A1

(c) $354 \leq d \leq 407$ B1 **(9)**

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4.

Stage	State	Action	Destination	Value	
1	*I*	*IL*	*L*	5*	
	J	*JL*	*L*	6*	A1
	K	*KL*	*L*	10*	
2	*F*	*FI* *FJ* *FK*	*I* *J* *K*	min(5, 5) = 5* min(2, 6) = 2 min(2, 10) = 2	
	G	*GI* *GJ* *GK*	*I* *J* *K*	min(8, 5) = 5 min(9, 6) = 6* min(3, 10) = 3	
	H	*HI* *HJ* *HK*	*I* *J* *K*	min(10, 5) = 5 min(2, 6) = 2 min(9, 10) = 9*	M1 A2
3	*B*	*BF* *BG* *BH*	*F* *G* *H*	min(8, 5) = 5 min(11, 6) = 6* min(4, 9) = 4	
	C	*CF* *CH*	*F* *H*	min(5, 5) = 5 min(10.5, 9) = 9*	
	D	*DF* *DH*	*F* *H*	min(9, 5) = 5 min(6, 9) = 6*	
	E	*EF* *EG* *EH*	*F* *G* *H*	min(12, 5) = 5 min(7, 6) = 6 min(15, 9) = 9*	M1 A1
4	*A*	*AB* *AC* *AD* *AE*	*B* *C* *D* *E*	min(1, 6) = 1 min(4.5, 9) = 4.5 min(13, 6) = 6 min(10, 9) = 9*	A1

giving route *AEHKL* M1 A1
shortest stage is 9 miles A1 **(10)**

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5. need to add dummy column giving M1

	19	69	168	0
	22	64	157	0
	20	72	166	0
	23	66	171	0
col min.	19	64	157	0

reducing rows will make no difference B1

reducing columns gives:

~~0~~	~~5~~	~~11~~	~~0~~	
~~3~~	~~0~~	~~0~~	~~0~~	
1	8	9	0	
4	2	14	0	

(N.B. a different choice of lines will lead to the same final assignment) M1 A1

3 lines required to cover all zeros, apply algorithm B1

0	5	11	1
~~3~~	~~0~~	~~0~~	~~1~~
0	7	8	0
3	1	13	0

M1 A1

3 lines required to cover all zeros, apply algorithm

0*	4	10	1
~~4~~	~~0~~	~~0*~~	~~2~~
~~0~~	~~6~~	~~7~~	~~0*~~
~~3~~	~~0*~~	~~12~~	~~0~~

A1

4 lines required to cover all zeros so allocation is possible B1

stage 1 is run by Alex
stage 2 is run by Suraj
stage 3 is run by Darren M1 A1 **(11)**
Leroy does not take part

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6. *(a)*

		Y		row minimum
		Y_1	Y_2	
X	X_1	$^-2$	4	$^-2$
	X_2	6	$^-1$	$^-1$
column maximum		6	4	

M1 A1

max (row min) = $^-1$ min (col max) = 4
max (row min) ≠ min (col max) ∴ no saddle point B1

(b) (i) let X play strategies X_1 and X_2 with proportions p and $(1 - p)$
expected payoff to X against each of Y's strategies:

Y_1 $^-2p + 6(1 - p) = 6 - 8p$
Y_2 $4p - (1 - p) = 5p - 1$ M1 A1

for optimal strategy $6 - 8p = 5p - 1$
$\therefore 13p = 7,\ p = \frac{7}{13}$

$\therefore$ X should play X_1 $\frac{7}{13}$ of time and X_2 $\frac{6}{13}$ of time M1 A1

(ii) let Y play strategies Y_1 and Y_2 with proportions q and $(1 - q)$
expected loss to Y against each of X's strategies:

X_1 $^-2q + 4(1 - q) = 4 - 6q$
X_2 $6q - (1 - q) = 7q - 1$ M1 A1

for optimal strategy $4 - 6q = 7q - 1$
$\therefore 13q = 5,\ q = \frac{5}{13}$

$\therefore$ Y should play Y_1 $\frac{5}{13}$ of time and Y_2 $\frac{8}{13}$ of time M1 A1

(c) value of game = $6 - (8 \times \frac{7}{13}) = 1\frac{9}{13}$ M1 A1 **(13)**

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7. *(a)*

	D	*E*	*F*	Available
A	20			20
B	10	5		15
C			25	25
Required	30	5	25	

M1 A1

no. of rows + no. of cols – 1 = 3 + 3 – 1 = 5
in this solution only 4 cells are occupied, less than 5 ∴ degenerate B1

(b) placing 0 in (3, 2) as it has lowest cost of unoccupied cells
taking $R_1 = 0$, $R_1 + K_1 = 13$ ∴ $K_1 = 13$ $R_2 + K_1 = 10$ ∴ $R_2 = ^-3$
$R_2 + K_2 = 9$ ∴ $K_2 = 12$ $R_3 + K_2 = 6$ ∴ $R_3 = ^-6$ M1 A2
$R_3 + K_3 = 8$ ∴ $K_3 = 14$

	$K_1 = 13$	$K_2 = 12$	$K_3 = 14$
$R_1 = 0$	(0)	11	14
$R_2 = ^-3$	(0)	(0)	12
$R_3 = ^-6$	15	(0)	(0)

improvement indices, $I_{ij} = C_{ij} - R_i - K_j$

∴ $I_{12} = 11 - 0 - 12 = ^-1$
$I_{13} = 14 - 0 - 14 = 0$
$I_{23} = 12 - (^-3) - 14 = 1$ M1 A1
$I_{31} = 15 - (^-6) - 13 = 8$

pattern not optimal as there is a negative improvement index B1

applying algorithm let $\theta = 5$, giving

	D	*E*	*F*
A	$20 - \theta$	θ	
B	$10 + \theta$	$5 - \theta$	
C			25

	D	*E*	*F*
A	15	5	
B	15		
C			25

M1 A1

this solution is also degenerate
place 0 in (3, 2) again
taking $R_1 = 0$, $R_1 + K_1 = 13$ ∴ $K_1 = 13$ $R_1 + K_2 = 11$ ∴ $K_2 = 11$
$R_2 + K_1 = 10$ ∴ $R_2 = ^-3$ $R_3 + K_2 = 6$ ∴ $R_3 = ^-5$ M1 A1
$R_3 + K_3 = 8$ ∴ $K_3 = 13$

	$K_1 = 13$	$K_2 = 11$	$K_3 = 13$
$R_1 = 0$	(0)	(0)	14
$R_2 = ^-3$	(0)	9	12
$R_3 = ^-5$	15	(0)	(0)

∴ $I_{13} = 14 - 0 - 13 = 1$
$I_{22} = 9 - (^-3) - 11 = 1$
$I_{23} = 12 - (^-3) - 13 = 2$ M1 A1
$I_{31} = 15 - (^-5) - 13 = 7$

all improvement indices are non-negative ∴ pattern is optimal B1

15 units from *A* to *D*, 5 units from *A* to *E*,
15 units from *B* to *D*, 25 units from *C* to *F* A1

total cost = (15 × 13) + (5 × 11) + (15 × 10) + (25 × 8) = £600 A1 **(18)**

Total **(75)**

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Performance Record – D2 Paper D

Question no.	1	2	3	4	5	6	7	Total
Topic(s)	TSP, shortcuts	game, formulate lin. prog.	TSP, nearest neighbour	dynamic prog., maximin	allocation, dummy	game	transport., n-w corner, stepping-stone, degeneracy	
Marks	6	8	9	10	11	13	18	75
Student								

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